Revue Internationale des Sciences de Gestion
ISSN: 2665-7473
Volume 8 : Numéro 3

Analyzing the impact of oil price fluctuations on the Moroccan stock market: A Multifractal Detrended Cross-Correlation Approach

Analyse de l'impact des fluctuations des prix du pétrole sur le marché boursier marocain : une approche par corrélation croisée détrendée multifractale

BENBACHIR Soufiane
PhD, Chercheur associé au LERSG
Faculté des Sciences Juridiques, Economiques et Sociales-Agdal
Université Mohammed V-Rabat
Laboratoire d'études et de recherches en sciences de gestion (LERSG)

Date submitted : 13/05/2025

Date of acceptance : 24/07/2025

To cite this article :

BENBACHIR S. (2025) «Analyzing the impact of oil price fluctuations on the Moroccan stock market: A Multifractal Detrended Cross-Correlation Approach», Revue Internationale des Sciences de Gestion « Volume 8 : Numéro 3 » pp : 155 - 180

Abstract
This study investigates the multifractal characteristics of cross-correlations between the Brent crude oil market and the Moroccan stock market, focusing on both the overall MASI index and its sector-specific MASI Oil index, using the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA). Initial findings from the Cross-Correlation Significance Test and the DCCA Cross-Correlation Coefficient indicate persistent cross-correlations between the Brent crude oil market and both the MASI and MASI Oil indices, suggesting that shocks in the Brent market may exert lasting effects on these indices. These findings challenge the assumptions of market independence under the Efficient Market Hypothesis (EMH). Further analysis using MF-DCCA components, including the Generalized Hurst exponents, Rényi exponents, and Hölder Singularity Spectrum, reveals long-range persistent cross-correlations and multifractal characteristics in both pairs. Notably, the Brent–MASI pair displays stronger multifractality, indicating more persistent and potentially more volatile dynamics. Moreover, surrogate and shuffling transformations revealed that the multifractal nature of these cross-correlations is driven by long-term dependencies and heavy-tailed distributions. This study demonstrates the effectiveness of multifractal models in capturing the complex, non-linear, and long-range cross-correlations between oil prices and the Moroccan stock market. Nonetheless, the absence of high-frequency data limits the ability to analyze short-term market fluctuations.
Keywords : Multifractality, Cross-Correlation, Generalized Hurst Exponents, Rényi Exponents, Singularity Spectrum
JEL Classification : C10, C22, C58, C63, G10, G15

Résumé
Cette étude examine les caractéristiques multifractales des corrélations croisées entre le marché du pétrole brut Brent et le marché boursier marocain, en se concentrant sur l'indice global MASI et l'indice sectoriel MASI Oil, via l'Analyse de Corrélation Croisée Detrendée Multifractale (MF-DCCA). Les résultats initiaux du Test de Signification de Corrélation Croisée et du Coefficient de Corrélation Croisée DCCA révèlent des corrélations croisées persistantes entre le Brent et les indices MASI, suggérant des effets durables des chocs pétroliers. Ces résultats remettent en cause l'Hypothèse du Marché Efficace (EMH). L'analyse MF-DCCA avec les exposants de Hurst généralisés, de Rényi, et le Spectre de Hölder, montre des corrélations persistantes à long terme et des propriétés multifractales dans les deux paires, avec une multifractalité plus marquée pour la paire Brent–MASI, indiquant une dynamique plus volatile. Les transformations par randomisation et permutation confirment que cette multifractalité est due à des dépendances à long terme et à des distributions à queues lourdes. L'étude démontre l'efficacité des modèles multifractals pour représenter les relations complexes entre les prix du pétrole et le marché marocain, apportant des insights utiles pour la gestion des risques, la diversification des portefeuilles et les décisions politiques. L'absence de données à haute fréquence limite cependant l'analyse des fluctuations à court terme.
Mots clés : Multifractalité, Corrélation croisée, Exposants de Hurst généralisés, Exposants de Rényi, Spectre de singularité
JEL Classification : C10, C22, C58, C63, G10, G15

Introduction

The relationship between oil prices and stock markets has been a key area of interest for economists and financial analysts, given the critical role that oil plays in the global economy. Oil price fluctuations are often linked to macroeconomic variables, such as inflation, exchange rates, and industrial production, all of which can influence stock market performance. The impact of oil price changes on stock markets, particularly in oil-importing or oil-dependent countries, is well-documented in the literature, but the complexity of these interactions calls for more advanced methodologies to capture their dynamic and multifaceted nature.

In recent years, Morocco, an emerging economy in North Africa, has shown increasing sensitivity to global energy prices due to its dependence on oil imports for energy production and transportation. Morocco is heavily reliant on imports to meet its energy needs, making Brent Crude Oil a critical factor in the country's energy pricing and economic stability. Since Brent serves as a global benchmark for crude oil prices, fluctuations in its price directly impact Morocco's energy costs, particularly for refined products like gasoline and diesel. This reliance makes the Moroccan economy sensitive to changes in Brent prices, which can influence inflation, the balance of trade, and overall economic growth.

The relationship between Morocco and Brent Crude Oil is also significant in sectors like transportation, manufacturing, and agriculture, where fuel prices play a crucial role in determining costs. Given that Morocco imports the majority of its petroleum products, shifts in Brent prices can lead to changes in domestic fuel prices, affecting both consumers and industries. Additionally, Morocco's stock market, particularly indices like the MASI and the MASI Oil & Gas Index, may reflect the impact of Brent Crude Oil price movements, with energy companies and oil-related sectors showing correlations with global oil prices.

The Moroccan stock market, represented by the Casablanca Stock Exchange (CSE), plays a significant role in the national economy, and understanding the influence of oil price movements on this market is critical for policymakers, investors, and portfolio managers. Traditional approaches to analyzing the oil-stock market relationship often assume linearity and do not account for the intricate interdependencies between oil prices and stock indices, which can exhibit non-linear behavior and long-range dependencies. This raises an important question: To what extent do Brent crude oil price fluctuations influence the Moroccan stock market, particularly its oil-related sector, and how can these relationships be captured using more sophisticated, non-linear analytical methods?

To address these limitations, this study adopts the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA) approach, a powerful tool for uncovering non-linear relationships and multifractal properties within financial time series. Unlike standard correlation methods, MF-DCCA allows for the identification of long-term cross-correlations across different time scales, making it particularly useful for analyzing the complex interactions between oil prices and stock markets. This method has been successfully applied in various financial markets to investigate market efficiency, volatility, and interdependence, but its application to the Moroccan stock market, especially in the context of oil price fluctuations, remains underexplored.

The objective of this paper is to analyze the impact of Brent crude oil prices on the Moroccan stock market, with a specific focus on the oil sectorial index of the Casablanca Stock Exchange. By employing the MF-DCCA method, we aim to uncover the multifractal cross-correlations between the two markets, shedding light on the degree of dependence and persistence that exists between them. Using daily data on Brent crude oil prices and the MASI Oil & Gas Index from the Casablanca Stock Exchange, the MF-DCCA method is applied to measure the strength and persistence of multifractal cross-correlations between the two series. This approach enables a more nuanced understanding of how global oil price movements may influence sector-specific market behavior in Morocco. Our findings will contribute to the growing body of literature on oil-stock market interdependencies and provide valuable insights for investors and policymakers in Morocco, especially in terms of risk management, portfolio diversification, and economic policy design.

The structure of the paper is as follows: Section 1 reviews the existing literature. Section 2 details the data and methodology employed. Section 3 presents the results along with a discussion of the findings. Lastly, Section 4 concludes the study and outlines its implications.

1. Literature review

The relationship between oil price shocks and macroeconomic variables has drawn considerable scholarly interest, especially due to oil's central role in global economic dynamics. Traditional econometric models—such as VAR, VECM, DCC-GARCH-GJR, and structural VAR—have been widely used to investigate the transmission of oil price volatility to indicators like GDP growth, inflation, and trade balances. Within this framework, several studies have focused specifically on the Moroccan economy, offering valuable regional insights that enrich the broader discourse. Notably, Azami (2013) examined the relationship between oil price volatility and economic growth in Morocco using semi-annual data from 1980 to 2013. By testing symmetrical, asymmetrical, and context-sensitive hypotheses through econometric models, this study reveals how oil price changes may impact GDP growth differently depending on economic conditions. Complementing this work, Misry 2020) conducted an empirical evaluation of the impact of rising oil prices on Morocco's trade balance and inflation. Using a Vector Autoregression (VAR) model, the study found that oil price increases deteriorate Morocco's trade balance, but—perhaps counterintuitively—do not lead to inflation. These finding challenges conventional expectations, suggesting that factors such as government subsidies, monetary policy, or import structures may cushion the inflationary effects of oil price hikes in the Moroccan context.

In parallel, multifractal methods (e.g., MF-DCCA) offer complementary insights by capturing nonlinearities and scale-dependent behaviors in financial markets. Among the first studies to adopt multifractal techniques, Burugupalli (2015) used MF-DCCA to examine cross-correlations between gold and WTI crude oil, revealing strong multifractal correlations in the short term, which weakened in the long term. Similarly, Yao et al. (2020) applied MF-DCCA to analyze the nonlinear relationships between economic policy uncertainty, crude oil, and stock markets, highlighting persistent multifractal correlations across time. In addition, Adekoya et al. (2023) explored the effects of the Russia-Ukraine war and found that oil price shocks exhibited stronger multifractal behavior, particularly in European stock markets.

Moreover, Jianfeng et al. (2016) demonstrated the usefulness of MF-DCCA in analyzing cross-correlations between crude oil and exchange rates, capturing multifractal behaviors often overlooked by traditional econometric models. This evidence suggests that multifractal methods offer richer insights into the dynamic and evolving relationships in financial markets.

In contrast to multifractal approaches, traditional econometric models have been widely used to examine oil-stock market dynamics. For example, Cuñado and Pérez de Gracia (2014) used VAR and VECM models to analyze the effects of oil price shocks on stock returns in oil-importing European economies. They concluded that supply-side shocks were more influential than demand-side shocks, negatively impacting stock returns.

Similarly, Filis et al. (2011) employed DCC-GARCH-GJR models to study the time-varying relationship between oil prices and stock markets. Their results indicated that precautionary demand shocks led to negative correlations, while aggregate demand shocks resulted in positive ones. Likewise, Fang and You (2014) used a structural VAR model to study the effects of oil price shocks on stock markets in China, India, and Russia, revealing that the impact of oil price shocks varied depending on each country's economic structure and oil dependency.

Furthermore, Gomez-Gonzalez et al. (2020) explored interconnectedness between Brent crude oil prices and stock market indices in oil-dependent economies using LASSO methods, particularly during financial distress. These studies collectively underline the importance of traditional econometric approaches in understanding the oil-stock market nexus, especially in terms of causality and volatility transmission. Similarly, Hwang and Kim (2021) found that oil price shocks, particularly demand-driven ones, had stronger and more persistent effects on stock markets during recessions.

In addition to econometric models, wavelet and causality-based approaches have also been employed to study the oil-stock market relationship. For instance, Jammazi et al. (2017) used wavelet analysis and dynamic causality tests to examine time-varying causal relationships between oil prices and stock returns. Their findings showed that bidirectional causality intensified during crises, such as the Global Financial Crisis (GFC).

Similarly, Khalfaoui et al. (2019) studied volatility spillovers between oil and stock markets in oil-importing and oil-exporting countries. They found that oil-importing nations were more susceptible to lagged oil price shocks, emphasizing the importance of studying regional disparities in oil-stock market correlations.

Building on the broader market analyses, several studies have focused on industry-specific effects of oil price shocks. Nandha and Faff (2008) analyzed 35 global industry indices and concluded that rising oil prices typically hurt stock returns, except in oil-related industries. In a similar vein, Rahman (2022) used a nonlinear bivariate SVAR model to explore the asymmetric relationship between oil price shocks and U.S. stock returns, with results suggesting that oil price volatility plays a crucial role in shaping firms' investment decisions.

These findings highlight how oil price shocks do not affect all industries uniformly, and sector-specific studies provide critical insights for investors and policymakers aiming to mitigate risk.

Turning to volatility and correlation forecasting, Ivanovski and Hailemariam (2021) applied the Generalized Autoregressive Score (GAS) model to forecast volatility and correlation between oil prices and stock returns. Their findings showed that correlations varied significantly during periods of economic distress. In addition, Ben-Salha and Mokni (2022) combined Detrended Cross-Correlation Analysis (DCCA) with a quantile-based approach, showing that cross-correlations between WTI crude oil and the S&P 500 index were highly sensitive to market conditions.

Several studies have continued to advance our understanding of the oil-stock market relationship. For example, Yang et al. (2019) used the DCC–MIDAS method to explore long-term correlations between crude oil and stock markets, revealing a positive long-term correlation influenced by macroeconomic factors such as the risk-free rate, economic activity, and credit risk.

Similarly, Maghyereh and Hussein (2022) applied MFCCA to analyze the impact of the COVID-19 pandemic on US stock markets and oil prices, finding that cross-correlations strengthened during the pandemic, particularly in the long term.

On the other hand, Nakhipbekova et al. (2020) used a multiple structural break panel cointegration test to investigate long-term relationships between energy prices and stock markets in developed countries, while Escribano et al. (2023) applied a Dynamic Conditional Correlation Skew Student Copula model to reveal that oil acted as a hedge for oil-importing nations and a diversifier for exporting countries.

Wątorek et al. (2019) employed multifractal cross-correlation analysis to explore correlations between WTI crude oil futures and currencies, particularly during oil market bear phases, while Fernández and Roberto (2021) emphasized the significance of oil prices in developing risk-hedging strategies for investors in Mexico.

Finally, Barragan et al. (2013) used wavelet-based methods to analyze oil shocks and stock market crashes, showing that correlations between oil and stock markets turned negative during oil shocks but were positive or near-zero during stable periods. Similarly, Mhadhbi and Guelbi (2024) explored volatility transmission from Brent oil prices to MENA stock markets, identifying strong volatility persistence in Tunisia and Oman, while Morocco showed resilience.

In conclusion, both multifractal and traditional econometric approaches provide valuable insights into the complex relationship between oil price shocks and stock markets. Multifractal methods, in particular, allow for a deeper understanding of the persistence and nonlinear interactions that characterize this relationship. Additionally, these methods offer important implications for risk management and investment strategies, especially in emerging markets like Morocco. By capturing scale-dependent correlations and the evolving nature of oil-price impacts on stock returns, multifractal approaches enhance the accuracy of forecasts and the formulation of effective investment strategies.

2. Data and methodology

2.1 Data

The data consists of daily closing prices of Brent Oil Futures (BRENT), MASI index and MASI Oil & Gas sectorial index (MASI Oil). The data span from 04/01/2010 to 17/01/2025, comprising nearly 3915 observations. All data were downloaded from the website www.investing.com. The prices were then converted into logarithmic returns:

$$r_t = ln\left(\frac{P_t}{P_{t-1}}\right) = ln(P_t) - ln(P_{t-1}) \quad (1)$$

where P_t denotes the index daily price and ln corresponds to the natural logarithm.

2.2. Methodology

2.2.1 Multifractal Detrended Cross-Correlation analysis

In this section, we introduce the Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), developed by Zhou (2008). This method integrates the Detrended Cross-Correlation Analysis (DCCA) introduced by Podobnik and Stanley (2008) with the Multifractal Detrended Fluctuation Analysis (MF-DFA) proposed by Kantelhardt et al. (2002). MF-DCCA is designed to examine the multifractal characteristics of the cross-correlation between two time series. The method involves five steps, which are outlined below.

Consider two time series, $x(k)$ and $y(k)$, where k ranges from 1 to N, with N representing the length of the series. It is assumed that these time series have limited non-zero values, meaning that $x(k) = 0$ and $y(k) = 0$ for only a minimal fraction of the time points k.

Step 1: In this first step, we determine the profiles $X = \left(X(i)\right)_{1 \le i \le N}$ and $Y = \left(Y(i)\right)_{1 \le i \le N}$ of the series $x = \left(x(k)\right)_{1 \le k \le N}$ and $y = \left(y(k)\right)_{1 \le k \le N}$:

$$X(i) = \sum_{k=1}^{N} (x(k) - \bar{x}) \quad (2)$$

$$Y(i) = \sum_{k=1}^{N} (y(k) - \bar{y}) \quad (3)$$

where $\bar{x}$ and $\bar{y}$ are the means of the series $\left(x(k)\right)_{1 \le k \le N}$ and $\left(y(k)\right)_{1 \le k \le N}$ defined by:

$$\bar{x} = \frac{1}{N} \sum_{k=1}^{N} x(k) \quad (4)$$

$$\bar{y} = \frac{1}{N} \sum_{k=1}^{N} y(k) \quad (5)$$

Step 2: For a given time scale s such that $10 \leq s \leq \mathrm{N}/3$, we divide the profiles X and Y into $N_s = Int(N/s)$ non-overlapping segments of the same length s, where $Int(.)$ represents the function that gives the integer part of a real number. Since N is generally not a multiple of s, a short part at the end of the profiles may be neglected. To incorporate all the ignored parts of the series, the same procedure is repeated starting from the end of the profile. Thus, we obtain $2N_s$ segments.

We have two types of segmentation for $1 \leq v \leq N_s$ and $Ns + 1 \leq v \leq 2N_s$:

- For $1 \leq v \leq N_s$ we use $X\big((v-1)s+1\big) \cdots X\big((v-1)s+s\big)$ and $Y\big((v-1)s+1\big) \cdots Y\big((v-1)s+s\big)$.
- For $N_s + 1 \leq v \leq 2N_s$ we use $X\big((N-v-N_s)s+1\big) \cdots X\big((N-v-N_s)s+s\big)$ and $Y\big((N-v-N_s)s+1\big) \cdots Y\big((N-v-N_s)s+s\big)$.

Step 3: In each segment, we use the Ordinary Least Squares (OLS) method to properly fit data in each segment with a local trend. We denote by $p^m_{X,v}(i)$ and $p^m_{Y,v}(i)$ the fitting polynomials of order m for respectively the profile X and the profile Y for the v-th segment. For $1 \leq v \leq 2N_s$, we set:

$$p^m_{X,v}(i) = \alpha^v_0 + \alpha^v_1 . i + \cdots + \alpha^v_m . i^m \quad (6)$$
$$p^m_{Y,v}(i) = \beta^v_0 + \beta^v_1 . i + \cdots + \beta^v_m . i^m \quad (7)$$

In the empirical study, the order m of the fitting polynomial can be linear, quadratic, cubic, or even of a higher order. Choosing an appropriate value of m can avoid overfitting the series.

Step 3: After determining the fitting polynomial $p^m_{X,v}(i)$ and $p^m_{Y,v}(i)$, we calculate the detrended covariances $f^2_{X,Y}(v,s)$ for all time scales s and for every segment $1 \leq v \leq 2N_s$:

- For $1 \leq v \leq N_s$, the covariance $f^2_{X,Y}(v,s)$ is defined by:

$$f^2_{XY}(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left|X\big((v-1)s+i\big) - p^m_{X,v}(i)\right| . \left|Y\big((v-1)s+i\big) - p^m_{Y,v}(i)\right| \quad (8)$$

- For $N_s \leq v \leq 2N_s$, the covariance $F^2_{X,Y}(v,s)$ is defined by:

$$f^2_{XY}(v,s) = \frac{1}{s}\sum_{i=1}^{s}\left|X\big((N-v-N_s)s+i\big) - p^m_{X,v}(i)\right| . \left|Y\big((N-v-N_s)s + i\big) - p^m_{Y,v}(i)\right| \quad (9)$$

Step 4: By averaging the covariances over all segments, we obtain the fluctuation functions $F^{XY}_q(s)$ of order q defined by:

- For $q \neq 0$:

$$F_q^{XY}(s) = \left[\frac{1}{2N_s}\sum_{v=1}^{2N_s}\left(f_{XY}^2(v,s)\right)^{\frac{q}{2}}\right]^{\frac{1}{q}} \quad (10)$$

- For $q = 0$:

$$F_0^{XY}(s) = exp\left[\frac{1}{4N_s}\sum_{v=1}^{2N_s} ln\left(f_{XY}^2(v,s)\right)\right] \quad (11)$$

The purpose of the MF-DCCA procedure is primarily to determine the behavior of the fluctuation functions $F_q^{XY}(s)$ as functions of the time scale s for various values of q. To this end, steps 2 through 4 must be repeated for different time scales s.

Step 5 : We analyze the multi-scale behavior of the fluctuation functions $F_q^{XY}(s)$ by estimating the slope of the log-log plots of $F_q^{XY}(s)$ versus s for different values of q. If the analyzed time series X and Y exhibits long-range cross-correlation according to a power-law, such as fractal properties, the fluctuation function $F_q^{XY}(s)$ will behave, for sufficiently large values of s, according to the following power-law scaling law:

$$F_q^{XY}(s) \sim s^{H_{XY}(q)} \quad (12)$$

or

$$log\left(F_q^{XY}(s)\right) = log\left(s^{H_{XY}(q)}\right) + log(C) \quad (13)$$

where $H_{XY}(q)$ is called the generalized Hurst exponent, which is the power-law cross-correlation of the two series X and Y.

When $H_{XY}(q)$ depend on q, the cross-correlation of the two-time series is multifractal, otherwise it is monofractal.

To estimate the values of $H_{XY}(q)$ for different values of q, we perform a semi-logarithmic regression of the time series $H_{XY}(q)$ on the time series $F_q^{XY}(s)$. When $q = 2$, $H_{XY}(2)$ is known as the standard Hurst exponent. When $H_{XY}(2) = 0.5$, there are no cross-correlations. When $H_{XY}(2) > 0.5$, the cross-correlations are long-range persistent, while $H_{XY}(2) < 0.5$, the two series have long-range ant-persistent cross-correlations. In addition, for positive q, $H_{XY}(q)$ describes the scaling behavior of intervals with large fluctuations. On the contrary, for negative q, $H_{XY}(q)$ describes the scaling behavior of segments with wavelet fluctuations.

$H_{XY}(q)$ is a decreasing function and to measure the degree of multifractality between the two series can be measured by the variation ΔH_{XY} between the minimum and maximum values as defined below:

$$\Delta H_{XY} = H_{XY-Max} - H_{XY-Min} = H_{XY}(q_{min}) - H_{XY}(q_{max}) \quad (14)$$

The larger ΔH_{XY} is, the stronger the degree of multifractality will be.

For positive values of q, the average fluctuation function $F_q^{XY}(s)$ is dominated by segments v with large covariances $f_{XY}^2(v,s)$. Thus, for $q > 0$, the generalized Hurst exponents $H_{XY}(q)$ describe the scaling properties of large fluctuations. In contrast, for $q < 0$, the exponents $H_{XY}(q)$ describe the scaling properties of small fluctuations.

It is well known that the generalized Hurst exponent $H_{XY}(q)$ defined by the MF-DCCA method is directly related to the multifractal scaling exponent $\tau_{XY}(q)$, commonly known as the Rényi exponent:

$$\tau_{XY}(q) = q.H_{XY}(q) - 1 \quad (15)$$

If the Rényi exponent $\tau_{XY}(q)$ increase nonlinearly with q, the cross-correlation of the two series is multifractal. Otherwise, if the Rényi exponent $\tau_{XY}(q)$ is a linear function of q, then the cross-correlation is monofractal.

Another interesting way to characterize the multifractality of the time series cross-correlations, is to use the Hölder spectrum or the singularity spectrum $f_{XY}(\alpha_{XY})$ of the Hölder exponent α_{XY}. It is well known that the singularity spectrum $f_{XY}(\alpha_{XY})$ is related to the Rényi exponent $\tau_{XY}(q)$ through the Legendre transform:

$$\begin{cases} \alpha_{XY} = \tau'_{XY}(q) \\ f_{XY}(\alpha_{XY}) = q.\alpha_{XY} - \tau_{XY}(q) \end{cases} \quad (16)$$

where $\tau'_{XY}(q)$ is the derivative of the function $\tau_{XY}(q)$.

The Hölder exponent α_{XY} characterizes the intensity of the singularity, and the singularity spectrum $f_{XY}(\alpha_{XY})$ represents the Hausdorff dimension of the fractal subset with exponent α_{XY}.

When the cross-correlation between the two series is multifractal, then the singularity spectrum $f_{XY}(\alpha_{XY})$ present a concave bell-shaped curve.

The richness of the multifractality can be determined by the width of the spectrum defined by:

$$\Delta\alpha_{XY} = \alpha_{XY-max} - \alpha_{XY-min} \quad (17)$$

Thus, the wider the spectrum, the richer the multifractal behavior of the cross-correlation of the analyzed time series.

We can easily deduce the relationship between the generalized Hurst exponent $h(q)$ and the singularity spectrum $f_{XY}(\alpha_{XY})$:

$$\begin{cases} \alpha_{XY} = H_{XY}(q) + q.H'_{XY}(q) \\ f_{XY}(\alpha_{XY}) = q.\left(\alpha_{XY} - H_{XY}(q)\right) + 1 \end{cases} \quad (18)$$

2.2.2 Cross-correlation significance test

As a preliminary analysis, it is useful to examine the existence of cross-correlations qualitatively. To this end, Podobnik et al. (2009) developed the Q_{CC} statistic test.

Suppose $(X_t)_{1\leq t\leq N}$ and $(Y_t)_{1\leq t\leq N}$ are two time series of length N. Podobnik et al. (2009) defined the cross-correlation function C_i by : for $1 \leq i \leq N-1$

$$C_i = \frac{\sum_{k=i+1}^{N} X_k . Y_{k-1}}{\sqrt{\sum_{k=1}^{N} X_k^2 \cdot \sum_{k=1}^{N} Y_k^2}} \tag{19}$$

The cross-correlation statistic Q_{CC} is defined by : for $1 \leq s \leq N-1$

$$Q_{CC}(s) = N^2 . \sum_{i=1}^{s} \frac{C_i^2}{N-s} \tag{20}$$

Podobnik et al. (2009) demonstrated that $Q_{CC}(s)$ is approximately $\chi^2(s)$ distributed with s degrees of freedom. The test can be used to test the null hypothesis that none of the first s cross-correlation coefficients is different from zero. The authors proposed to use the statistic by plotting the test statistic $Q_{CC}(s)$ versus the critical values $\chi^2(s)$ for a broad range of the degree of freedom s. If for a broad range of s the test statistic $Q_{CC}(s)$ exceeds the critical values at a 95% level of confidence, we can claim that cross-correlations are not only significant, but there are long-range cross-correlations. However, this test statistic, as a correlation coefficient, is a measure of linear cross-correlations, and as pointed by Podobnik et al. (2009) , this cross-correlations test should be used to test the presence of cross-correlations only qualitatively.

2.2.3 DCCA Cross-Correlation Coefficient

Based on the Detrended Cross Correlation analysis (DCCA) and the detrended fluctuation analysis (DFA), Zebende (2011) proposed a DCCA cross-correlation coefficient to quantify the cross-correlation between two non-stationary series. Using the previous notations of the MF-DCCA method, the DCCA cross-correlation coefficient $\rho_{DCCA}(s)$ is defined as the ratio of the detrended covariance function between X and Y to the product of the detrended variance functions of X and Y:

$$\rho_{XY}(s) = \frac{F_{DCCA}^2(s)}{\sqrt{F_{DFA-X}^2(s)} \times \sqrt{F_{DFA-Y}^2(s)}} \tag{14}$$

where $F_{DFA-X}^2(s)$, $F_{DFA-Y}^2(s)$ and $F_{DCCA}^2(s)$ are defined by replacing q by 2 in formula (10):

$$F_{DFA-X}^2(s) = \left(F_2^{XX}(s)\right)^2 \qquad F_{DFA-Y}^2(s) = \left(F_2^{YY}(s)\right)^2 \qquad F_{DCCA}^2(s) = \left(F_2^{XY}(s)\right)^2 \tag{15}$$

The cross-correlation coefficient $\rho_{XY}(s)$ is an effective measure with properties like those of the standard correlation coefficient. It is a dimensionless quantity that ranges from -1 to 1. When $\rho_{XY}(s) = 0$, there is no cross-correlation between the two series. A value of $-1 < \rho_{XY}(s) < 0$ indicates an anti-persistent cross-correlation, while $0 < \rho_{XY}(s) \leq 1$ suggests a persistent cross-correlation between the two series. If $\rho_{XY}(s) = -1$, the two series are perfectly anti-persistent cross-correlated. Conversely, if $\rho_{XY}(s) = 1$, the two series are perfectly persistent cross-correlated.

2.2.4 Sources of cross-correlation multifractality

It is well known that two primary sources of cross-correlation multifractality in bivariate time series are long-term temporal cross-correlations and heavy-tailed distributions.

To determine how each source contributes to the overall cross-correlation multifractality, we use two transformations on the two original return series, namely random permutation (shuffling) and phase randomization (surrogate).

By permuting the return series, the distribution of different moments is preserved, but long-term correlations are eliminated. After random permutation, the data have the same distribution but no temporal correlation or memory.

The phase randomization helps isolate the contribution of long-term correlations to multifractality by randomly shifting the temporal phases of the data and disrupting these correlations while preserving their overall fluctuation behavior.

In the literature, there are various techniques for phase randomization:

- Inverse Fast Fourier Transform (IFFT) (Proakis and Dimitris (1996).
- Iterated Algorithm (iAAFT) (Schreiber and Schmitz (1996)).
- Statically Transformed Autoregressive Process (STAP) (Kugiumtzis (2002))

All the techniques are based on the Fourier transform.

In this study, we used two shuffling techniques, based on two functions “randperm” and “randi” available in the software MATLAB. For the phase randomization we applied the Inverse Fast Fourier Transform (IFFT) method.

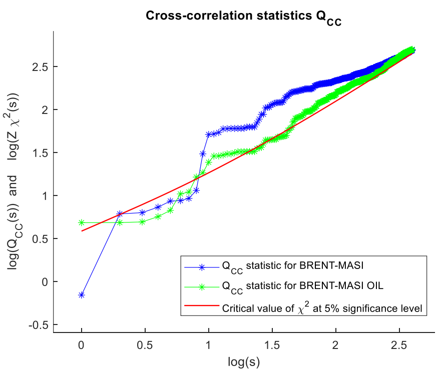
3. Results and discussion

3.1 Result of cross-correlation significance test

In this section, we have checked qualitatively the presence of the cross-correlations between the Masi index and the oil sectorial index with the Brent crude oil index, using the Q_{CC} statistic.

For the two pairs Brent-Masi and Brent-Masi Oil, we have plotted the decimal logarithm of the test statistic $Q_{CC}(s)$ versus the decimal logarithm of the critical values $\chi^2_{0,5}(s)$ at 5% significance level for a broad range of the degree of freedom s, $1 \leq s \leq 400$. The results are given in Figure 1.

Figure N°1: $Q_{CC}(s)$ and $Log(\chi^2_{0,5}(s))$ vs. $Log(s)$ for the pairs Brent-Masi and Brent-Masi Oil



Source: Our own elaboration

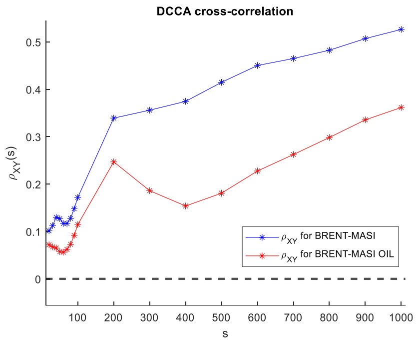
We can observe that for sufficiently large values of s, the $Q_{CC}(s)$ statistics exceed the critical values $\chi^2_{0,5}(s)$, indicating that the cross-correlations are significant for the Brent-Masi and Brent-Masi Oil. However, this test statistic is considered as a linear and qualitative measure of cross-correlations. The results must be confirmed by the application of MF-DCCA method.

3.2 Result of DCCA Cross-Correlation Coefficient

In this section, we have applied the DCCA cross-correlation coefficient to quantify the cross-correlation of the Brent crude oil with the Masi index and the Oil sectorial index.

The figure below shows the plots of the DCCA cross-correlation coefficient $\rho_{DCCA}(s)$ as a function of the variable s the two pairs Brent-Masi and Brent-Masi Oil.

Figure N°2 : DCCA cross-correlation coefficient $\rho_{XY}(s)$ vs. s for Brent-Masi and Brent-Masi Oil



Source: Our own elaboration

In the figure above, we observe that the two pairs Brent-Masi and Brent-Masi Oil show a DCCA cross-correlation with $0 < \rho_{DCCA}(s) < 1$, indicating persistent cross-correlation.

3.3 Application of MF-DCCA

In this section, we applied the MF-DCCA technique to analyze the multifractal cross-correlation of the bivariate logarithmic return series.

3.3.1 Multi-scale behavior of the cross-correlation fluctuation functions

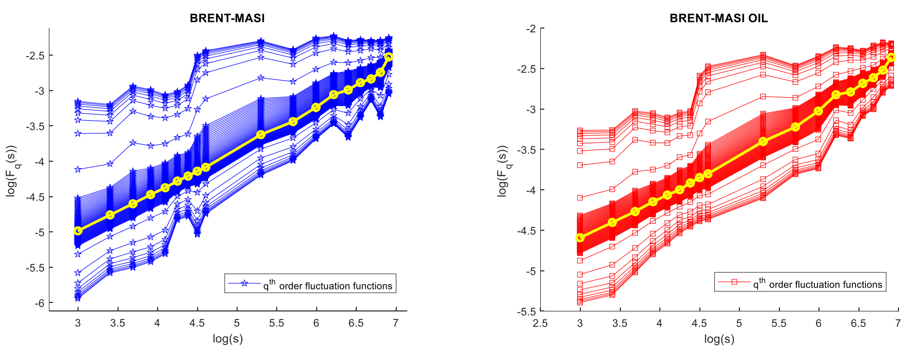
We analyzed the multi-scale behavior of the cross-correlation fluctuation functions $F_q^{XY}(s)$ with respect to the time scales s within the interval $[20{:}\,10{:}\,100{,}200{:}\,100{:}\,1000]$ for values of q in the interval $[-45{:}\,5{:}\,-5, -3.1{:}\,0.1{:}\,-0.1{,}0.1{:}\,0.1{:}\,3.1{,}5{:}\,5{:}\,45]$;

By regressing $Log(s)$ on $Log\left(F_q^{XY}(s)\right)$, we obtain an estimation of $H_{XY}(q)$ Brent-Masi and Brent-Masi Oil.

$$Log\left(F_q^{XY}(s)\right) \approx H_{XY}(q).Log(s) \tag{31}$$

The figure below shows the log-log plots of $Log\left(F_q^{XY}(s)\right)$ versus $Log(s)$.

Figure N°3: $Log\left(F_q^{XY}(s)\right)$ vs. $Log(s)$ for Brent-Masi and Brent-Masi Oil



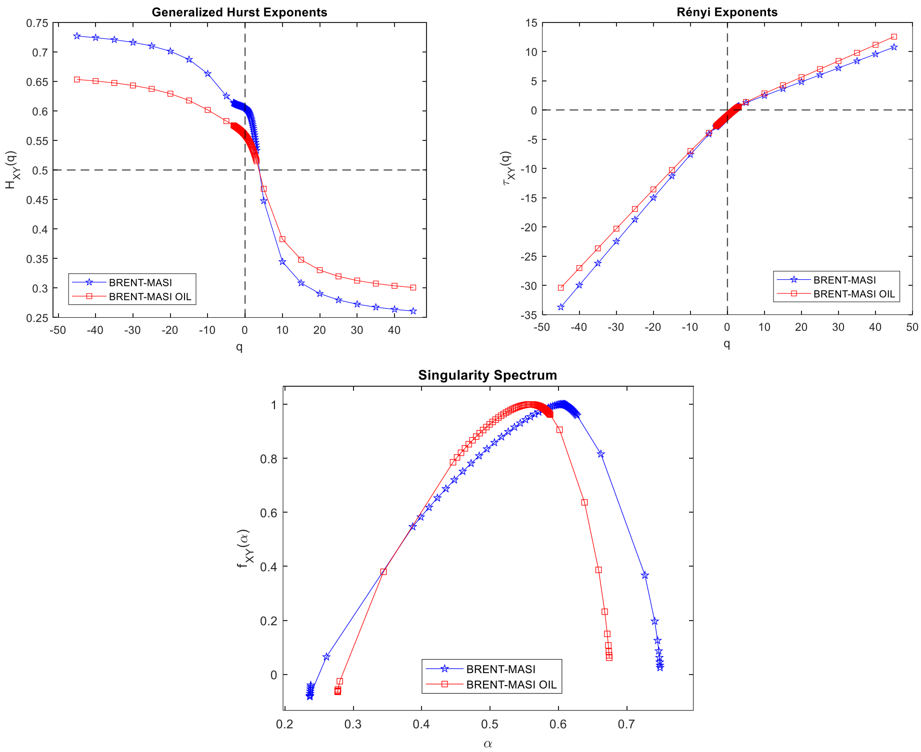
Source: Our own elaboration

As shown in Figure 3, the functions $F_q^{XY}(s)$ for the two pairs Brent-Masi and Brent-Masi Oil are increasing nonlinearly with s and with the change of q, which indicates a power-law relationship in the two pairs. The result shows the existence of long-range cross-correlation between the Brent and the MASI index and the Brent and the MASI Oil index. The presence of long-range cross-correlations indicates that market trends are persistent and interconnected, with shocks having prolonged effects across indices. These findings indicate that shocks in one market can have prolonged effects on others, suggesting interconnectedness that challenges the assumptions of market independence under the Efficient Market Hypothesis (EMH).

3.3.2 Multifractality and persistence of the cross-correlations

Figure 4 below shows the plots of the generalized Hurst functions $H_{XY}(q)$, the Rényi Exponent $\tau_{XY}(q)$ and the singularity spectra $f_{XY}(\alpha)$ for Brent-Masi and Brent-Masi Oil.

Figure N°4: Plots of $H_{XY}(q)$, $\tau_{XY}(q)$ and $f_{XY}(\alpha)$ for Brent-Masi and Brent-Masi Oil



Source: Our own elaboration

Figure 4 shows that, as q increases from -45 to 45, the generalized Hurst exponent $H_{XY}(q)$ decreases non-linearly and the Rényi Exponent $\tau_{XY}(q)$ increases non-linearly for Brent-Masi and Brent-Masi Oil. This indicates that the cross-correlations between the Brent and the MASI index and the Brent and the MASI Oil index exhibit multifractal nature. We can also observe in Figure 4 that the curves of the singularity spectrum functions $f_{XY}(\alpha)$ have an inverted concave bell-shaped parabolic shapes, confirming the multifractal nature.

We can determine the nature of the persistence of the cross-correlations by using the values of $H_{XY}(2)$. The following table display the values of $H_{XY}(2)$ for each Brent-Masi and Brent-Masi Oil.

Table N°1: Values of $H_{XY}(2)$ for Brent-Masi and Brent-Masi Oil

Pair	$H_{XY}(2)$
Brent-Masi	0.577
Brent-Masi Oil	0.536

Source: Our own elaboration

We observe that the generalized exponents $H_{XY}(2)$ for the two pairs are we have $H_{XY}(2) > 0.5$, indicating the long-range persistent cross-correlations for these pairs.

The multifractal nature of cross-correlations, as indicated by the non-linear behavior of generalized Hurst and Rényi exponents and the parabolic singularity spectra, highlights several key implications for market participants. The multifractal behavior reflects the complexity of volatility patterns across different time scales, highlighting that risk and correlations are not constant over time, which has practical implications for market participants, particularly in relation to identifying arbitrage opportunities and developing more resilient risk management strategies. For investors, fluctuating dependencies across time scales increase market complexity and risk unpredictability, particularly in long-term and high-frequency strategies. This multifractality reduces the effectiveness of traditional risk models. Portfolio managers should use multifractal models to optimize asset allocation because these models account for time-varying correlations and market dynamics. Unlike traditional models, which assume constant correlations, multifractal models capture how correlations change with market conditions, helping managers make informed decisions. They also help in managing risk by modeling volatility at different time scales, adapting to market shocks, and accounting for extreme events. By considering multifractal behavior, managers can improve diversification, reduce risk, and enhance portfolio performance, especially in unpredictable market environments. Policymakers should consider the non-linear and scale-dependent effects of shocks and interventions, as they can have unpredictable impacts across interconnected markets. In risk management, the uneven distribution of risk across time scales requires more advanced, non-linear models to properly assess and manage market risks.

3.3.3 Assessing the strength of the multifractality

The strength of multifractality of the cross-correlations could be measured by the difference between the smallest and largest values of $H_{XY}(q)$:

$$\Delta H_{XY} = H_{XY-Max} - H_{XY-Min} = H_{XY}(q_{min}) - H_{XY}(q_{max}) \quad (32)$$

or by the width of the spectrum, given by:

$$\Delta\alpha_{XY} = \alpha_{XY-max} - \alpha_{XY-min} \quad (33)$$

The table below present the degrees of multifractality for the 10 pairs of indices based on ΔH_{XY} and $\Delta\alpha_{XY}$.

Table N°2: Degrees of multifractality of Brent-Masi and Brent-Masi Oil based on ΔH_{XY}, $\Delta\alpha_{XY}$

Pairs of indices	ΔH_{XY}	$\Delta\alpha_{XY}$
Brent-Masi	0.466	0.511
Brent-Masi Oil	0.353	0.397

Source: Our own elaboration

We observe that all degrees of multifractality are different from 0, indicating multifractal behavior in the cross-correlations of the two pairs. The Brent-Masi pair exhibits the highest degree of multifractality. The Brent-Masi pair, exhibiting the highest degree of multifractality, has significant implications for investors, portfolio managers, policymakers, and risk management. For investors, the strong long-term dependencies between these markets suggest more persistent and potentially volatile trends, requiring adjusted strategies. Portfolio managers should account for varying multifractality to optimize diversification and risk management. Policymakers must recognize that shocks or interventions in one market could have long-lasting effects on interconnected markets, necessitating regulations that address both immediate and extended impacts. For risk management, the non-linear dependencies in the Brent-Masi pair emphasize the need for advanced models that can more accurately predict risks and improve hedging strategies.

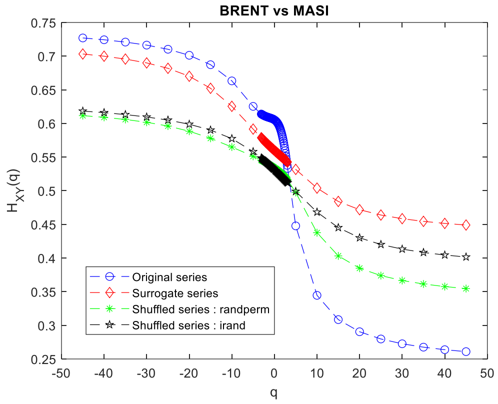
3.3.4 Source of multifractality

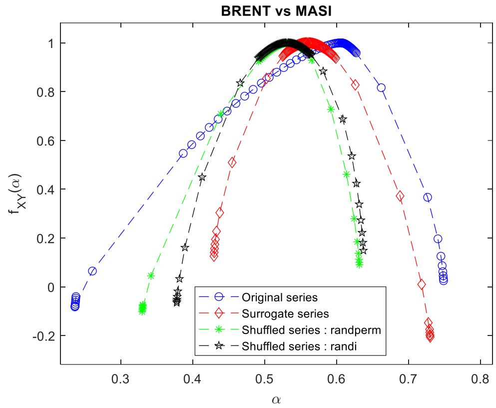
As previously noted, there are two different sources of multifractality, long-term temporal cross-correlations and the heavy tails distributions. To determine how each source contributes to the overall cross-correlations multifractality, we will use two transformations on the original logarithmic return series, the Shuffling and Phase randomization (Surrogation).

In this study, we used two shuffling techniques, namely “randperm” and “randi”. For phase randomization, we applied the Inverse Fast Fourier Transform (IFFT) method.

Figures 5 below compares the curves of the generalized Hurst exponent $H_{XY}(q)$ and the curves of the singularity spectra $f_{XY}(\alpha)$ for the two pairs Brent-Masi and Brent-Masi Oil with those of the surrogate and the shuffled series.

Figure N°5: Generalized Hurst exponent $H_{XY}(q)$ vs. q for Brent-Masi and Brent-Masi Oil of original, surrogate and shuffled series





Source: Our own elaboration

We can observe in the figure 5 that the application of the shuffling and the surrogate transformations have reduced the degrees of multifractality of the original series.

To precisely measure how much multifractality has been reduced, we calculate the values of $\Delta H_{XY} = H_{XY-max} - H_{XY-min}$ and $\Delta\alpha_{XY} = \alpha_{XY-max} - \alpha_{XY-min}$ for the two pairs. Table 4 below illustrates the results.

Table N°3: Degrees of multifractality of original, surrogate and shuffled series based ΔH_{XY} and $\Delta\alpha_{XY}$

	Original		Surrogate		Shuffled-randperm		Shuffled-randi	
Pair	ΔH_{XY}	$\Delta\alpha_{XY}$	ΔH_{XY}	$\Delta\alpha_{XY}$	ΔH_{XY}	$\Delta\alpha_{XY}$	ΔH_{XY}	$\Delta\alpha_{XY}$
Brent-Masi	0.466	0.511	0.254	0.300	0.257	0.301	0.217	0.259
Brent-Masi Oil	0.353	0.397	0.223	0.268	0.288	0.333	0.216	0.261

Source: Our own elaboration

The results indicate that $\Delta H_{XY-Originate} > \Delta H_{XY-Surrogate}$, and $\Delta\alpha_{XY-orginale} > \Delta h_{XY-shuffled}$ for the two pairs Brent-Masi and Brent-Masi Oil, as confirmed in the previous table. This indicates that the multifractality of the cross-correlations has been reduced by both the surrogate and shuffled transformations. We conclude that both long-term cross-correlations and heavy-tailed distributions contribute to the cross-correlations multifractal behavior of the 10 pairs of index returns.

In summary, based on generalized Hurst exponents and singularity spectrum, we found that both long-term cross-correlations and heavy-tailed distributions contribute to the multifractal behavior of the two pairs Brent-Masi and Brent-Masi Oil. This finding has significant implications for various market players. For investors, it means that while market trends tend to persist over time, the likelihood of extreme market events, like crashes or surges, is higher, requiring more robust risk management strategies. Portfolio managers must account for non-linear relationships between assets and prepare for extreme co-movements, adjusting their models to manage risk during both stable and crisis periods. Policymakers should recognize that interventions could have prolonged, unpredictable effects, potentially leading to systemic risks, and design regulations to address extreme events and market interconnectivity. For risk management, traditional models are insufficient, as they don't capture non-linear dependencies and tail risks, highlighting the need for advanced models like those incorporating multifractal analysis or tail risk measures to better assess extreme market risks and improve resilience.

3.4 Comparison with previous studies

In numerous studies, researchers have explored the relationship between oil prices and stock markets, providing valuable insights into market interactions using a variety of approaches. Yang et al. (2019), for instance, employed the DCC-MIDAS approach to demonstrate long-term positive correlations between crude oil prices and stock markets, primarily driven by macroeconomic factors. Our study, in contrast, applies Multifractal Detrended Cross-Correlation Analysis (MF-DCCA), which allows us to capture not only these long-term dependencies but also the multifractal nature of the cross-correlations between oil prices and stock indices. While Yang et al. identified stable long-term positive correlations, we find that the relationship is far more complex, exhibiting scale-dependent behavior and non-linear dynamics that traditional models fail to detect.

Similarly, Maghyereh and Hussein (2022) used Multifractal Cross-Correlation Analysis (MFCCA) to study the effect of the COVID-19 pandemic on U.S. stock markets and oil prices, identifying stronger correlations during periods of crisis. Our results show that even in the absence of such a global crisis, cross-correlations between Brent crude oil prices and the Moroccan All Shares Index (MASI) are significant and persistent over time, suggesting that shocks in Brent crude oil market can lead to long-lasting effects in the stock markets.

Adekoya et al. (2023) also observed heightened multifractality in European stock markets in response to oil price shocks, similar to our findings of strong multifractal correlations between oil prices and the MASI Oil Index. While Adekoya et al. focused on the European context, our study expands these findings to the Moroccan market, suggesting that multifractality is not limited to developed markets and can be observed in emerging markets as well. The non-linear dynamics we detected mirror those reported by Jianfeng et al. (2016), who identified multifractal properties in the relationship between crude oil and exchange rates, further reinforcing the robustness of our multifractal approach.

From a risk management perspective, our findings align with the work of Cuñado and Pérez de Gracia (2014), who emphasized the significant impact of oil price fluctuations on stock returns in oil-importing economies. However, while their study identified this effect using traditional econometric models, we go further by demonstrating that these effects are multifractal in nature, and thus more complex than previously understood. Our analysis suggests that multifractal models, such as MF-DCCA, provide a more accurate representation of the time-varying, non-linear relationships between oil prices and stock indices, which are crucial for effective portfolio diversification and risk management.

Filis et al. (2011) found that the correlations between oil prices and stock markets vary depending on the type of oil price shock. Our study supports this finding but goes beyond it by showing that the nature of these correlations is also influenced by multifractal patterns that change over different time scales. This means that risk management strategies based on static models or simple shock classifications may be insufficient. Instead, multifractal models offer a more nuanced understanding of how oil price shocks impact stock returns, especially in emerging markets like Morocco.

In contrast to studies like those by Nakhipbekova et al. (2020), which used panel cointegration tests to explore long-term relationships between energy prices and stock markets, our approach reveals more detailed, multifractal interactions that cointegration tests may miss. Escribano et al. (2023), who employed a Dynamic Conditional Correlation Skew Student Copula model, found that oil can serve both as a hedge and a diversifier. Our results add another layer of complexity to this view, showing that oil's role as a hedge or diversifier is multifractal and varies depending on the time scale and market conditions.

Wątorek et al. (2019) applied MF-DCCA to study WTI crude oil futures and currencies during bear markets, finding significant multifractality. Our findings complement this by extending the analysis to Brent crude oil and stock indices, specifically in the Moroccan context, and confirming the presence of multifractal cross-correlations during periods of high volatility. However, while Wątorek et al. focused on bear markets, our study demonstrates that multifractality is a persistent feature of oil-stock market relationships, not limited to downturns.

Finally, Barragan et al. (2013) used wavelet-based research to show that oil shocks can negatively impact stock markets, with correlations turning positive during stable periods. Our results align with these findings but reveal that the correlation dynamics are more complex than a simple negative-to-positive shift. We observed that cross-correlations between Brent crude oil and MASI indices remain multifractal and time-varying, even during periods of market stability, suggesting that the relationship is more nuanced and multifaceted than previously thought.

In summary, while previous research has provided valuable insights into the correlations between oil prices and stock markets, our study contributes to the literature by showing that these relationships are multifractal, dynamic, and scale-dependent. Our results underscore the importance of using advanced multifractal models to fully capture the complexity of oil-stock market interactions, offering significant implications for portfolio management, risk assessment, and policymaking.

4. Conclusion

This study investigates the multifractal nature of cross-correlations between Brent crude oil prices and the MASI index, as well as between Brent and the MASI Oil index, using advanced statistical methods such as the MF-DCCA approach.

Initial results from the Cross-Correlation Significance Test and the DCCA Cross-Correlation Coefficient indicated persistent cross-correlations for the Brent-MASI and Brent-MASI Oil pairs, suggesting that shocks in the Brent market can have long-lasting impacts on these indices. These findings challenge the Efficient Market Hypothesis (EMH) by highlighting the existence of complex, non-linear interconnections between these markets. Further analysis using MF-DCCA tools—such as the Generalized Hurst exponents, Rényi exponents, and Hölder Singularity Spectrum—confirmed the presence of long-range persistent cross-correlations and multifractal behavior in both pairs. The multifractal properties, including power-law relationships in fluctuation functions, point to varying degrees of dependence across multiple time scales. Notably, the Brent-MASI pair exhibits stronger multifractality, indicating more persistent and potentially volatile trends.

Additionally, surrogate and shuffling transformations revealed that the multifractal nature of these markets is primarily influenced by long-term cross-correlations and heavy-tailed distributions, emphasizing the intricate volatility patterns observed across various time scales.
The time-varying correlations, particularly during periods of shocks and extreme events, highlight the importance of employing advanced multifractal models to optimize portfolio diversification, bolster risk management, and improve overall performance. These models offer a deeper understanding of volatility and risk across various time scales, making them crucial for managing risks in dynamic and volatile markets. Traditional models, which assume constant correlations, are insufficient in capturing the complex and evolving nature of these markets.
Moreover, the study presents important implications for policymakers, who must recognize that shocks in one market can trigger prolonged and unpredictable effects on interconnected markets. This underscores the need for regulations that account for both short-term and long-term impacts, considering the non-linear and scale-dependent nature of market dynamics.
One limitation of this study is the inability to apply the method to high-frequency data, as such data was not available. High-frequency data could provide more detailed insights into short-term market dynamics and improve the accuracy of multifractal analysis. The lack of access to this type of data limits the study's ability to capture finer market fluctuations and may affect the robustness of the results, particularly in highly volatile environments.
In conclusion, this study provides valuable insights into the complex relationships between oil prices and stock indices, emphasizing the importance of incorporating multifractality into market analysis. By adopting these advanced models, market participants can develop more robust strategies and make more informed decisions in an increasingly interconnected and volatile financial environment.

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